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Application of fuzzy-Bayes decision rule to the optical illusion state discrimination problem

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Abstract

We perform multi-dimensionalization of the mathematical model for the fuzzy-Bayes decision rule constructed by Hori and Tsubaki, and investigate its application to the optical illusion state discrimination problem. We first consider two mutually exclusive states of nature as true and false states, incorporating the concept of 0-1 loss as a fuzzy mode of statistical discriminate analysis, and apply the principle of fuzzy expected utility maximization to perform optical illusion state discrimination. We then propose the addition of new functions for transformation of the true state and the false state to the subjective prior distribution and the membership function, compute the fuzzy event probabilities, and select the state exhibiting high probability. It is also noted that these functions, unlike utility functions and loss functions, are not inverse functions and can be applied even if both are zero （indicating a visual hallucination state）

Keywords: Fuzzy-Bayes decision, optical illusion state, visual hallucination state, true state, false state

INTRODUCTION

The fuzzy theory framework was proposed by Zadeh (1965), as were the probability (1968) and possibility measures of fuzzy events (Zadeh 1977). Tanaka and Ishibuchi (1992), subsequently proposed max-product computation of the fuzzy event possibility measure. Its application to fuzzy event decision problems (the “fuzzy-Bayes decision rule”) was described by Uemura (1991), Uemura and Sakawa (1993), and Hori and Tsubaki (2014). Uemura (1991) showed that fuzzy inference is contained in subjective Bayesian inference, Uemura and Sakawa (1993) showed that fuzzy statistics is contained in subjective Bayesian statistics, and Hori and Tsubaki (2014) and Tsubaki and Hori (2016) showed that in utility （loss） function theory the Wald (1950) subjective modification is equivalent to fuzzy events. In the present paper, we first carry out multi-dimensionalization of the natural state in this fuzzy-Bayes decision rule and apply it to the optical illusion state discrimination problem. Because we are considering the optical illusion state, we treat the true state and the false state as fuzzy events with a real state and a real two-dimensional state of nature, and assume the distributions of these states are mutually exclusive and independent. We next map and transform the prior distributions using two transformation functions （true and false membership functions）, as the two-dimensional state of nature is intrinsically a single state and unlike true and false is not independently distributed. We then obtain the fuzzy event true and false probabilities and formulate the discrimination of the state having high probability. Lastly, we take the membership functions representing the true and false states as inversely related, as in the relationship between utility and loss functions, add the condition of orthogonality, and obtain the indiscriminate probability by solving the two-objective /programming problem of maximizing the probability representing the true state and minimizing the probability representing the false state. In comparison with other probabilities, this indiscriminate probability at its maximum is taken to represent the indiscriminate state. Fuzzy-Bayes decision rule multi-dimensionalization and application Uemura et al. (1991) and Hori and Tsubaki (2014) constructed a decision rule for fuzzy events following fuzzy event transformation by membership functions for natural states in no-data problems, and Tsubaki (2016) elucidated its relation to subjective Bayesian theory. In the present paper, we consider cases in which, for example, a person subject to hallucinations while viewing black and white paper strips claims that a strip that is actually white seems black. In such a case, it is natural to consider （multidimensionalize） fuzzy events involving two states such as white and black and mutually exclusive independent two-dimensional states of nature in terms of true and false states. Here, we assume that the membership functions Equations (1) and (2) for two two-dimensional fuzzy events F1 and F2 and a state of nature with the two-dimensional subjective prior distribution Equation (3) have been set by the decision maker, as in the method of Savage (1972) for lotteries.

$$\mu_{F_1}(s_1,s_2) = \mu_{F_1}(s_1) \wedge \mu_{F_1}(s_2) \tag{1}$$

$$\mu_{F_2}(s_1,s_2) = \mu_{F_2}(s_1) \wedge \mu_{F_2}(s_2) \tag{2}$$

$$\Pi(s_1,s_2) = \Pi(s_1) \times \Pi(s_2) \tag{3}$$

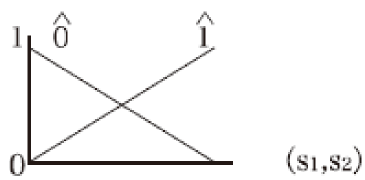
The two states of nature are then mutually exclusive and independent, and Equations. (1), (2) and (3), and thus hold. We next obtain the probability of the fuzzy events by the following equations. The Zadeh (1968) integral transform is applicable since we are here considering the case of a risk-neutral decision maker.

$$P(F_1) = \int \mu_{F_1}(s_1,s_2)\, \Pi(s_1,s_2)\, ds_1, ds_2 \tag{4}$$

$$P(F_2) = \int \mu_{F_2}(s_1,s_2)\, \Pi(s_1,s_2)\, ds_1, ds_2 \tag{5}$$

Lastly, because we are considering fuzzy events with real states, we can now perform the decision making with consideration of the following fuzzy 0-1 loss equation as an extension of ordinary decision analysis.

$$\begin{bmatrix} \hat{0} & \hat{1} \\ \hat{1} & \hat{0} \end{bmatrix}$$



(6)

We compute the fuzzy loss by Equations (7) and (8) using the integral transform, multiply by the fuzzy probability and compute the expected fuzzy loss, by Equations. (9) and (10), and take the minimum expected fuzzy loss as the optical illusion state discriminant.

$$L_{F_1} = \int L_{F_1}(s_1,s_2)\, \Pi(s_1,s_2)\, ds_1, ds_2 \tag{7}$$

$$L_{F_2} = \int L_{F_2}(s_1,s_2)\, \Pi(s_1,s_2)\, ds_1, ds_2 \tag{8}$$

$$E(F_1) = P(F_1) \times L_{F_1} + P(F_2) \times L_{F_2} \tag{9}$$

$$E(F_2) = P(F_1) \times L_{F_2} + P(F_2) \times L_{F_1} \tag{10}$$

It may be noted that if this fuzzy 0-1 loss is a type-2 fuzzy set, for example, in a case where identification is obtained by the Seo (2002) lottery method, then the upper and lower bounds may be taken to represent risk-tolerant and risk-averse conditions, respectively. It is then qualitatively preferable to use max-product computation for the upper bound, mini-max computation for the lower bound, and integral computation for centroid priority, as noted by Hori and Tsubaki (2014).

Fuzzy inference with a new membership function

Subjective prior distributions and membership functions are also necessary in fuzzy inferences for no-data problems. For this purpose, we add two new membership functions, the functions T（s） and F（s） which map and transform the state of nature to a true state and a false state, respectively, and consider the optical illusion state discrimination in one fuzzy event. Here we take the fuzzy event F3 having a real state as one. When we apply the extension principle of the fuzzy mathematics mapping by these two functions as described below, we obtain the membership function and the subjective prior distribution in the fuzzy event mapping and transformation by the following equations.

$$\mu_{\mathrm{T}}(\mathrm{x}) = \int \mu(\mathrm{s})/\mathrm{T}(\mathrm{s}) \triangleq \sup_{x=\mathrm{T}(\mathrm{s})} \mu(\mathrm{s}) = \mu(\mathrm{T}^{-1}(\mathrm{x})) \tag{11}$$

$$\Pi_{\mathrm{T}}(\mathrm{x}) = \int \Pi(\mathrm{s})/\mathrm{T}(\mathrm{s}) \triangleq \sup_{x=\mathrm{T}(\mathrm{s})} \Pi(\mathrm{s}) = \Pi(\mathrm{T}^{-1}(\mathrm{x})) \tag{12}$$

$$\mu_{\mathrm{F}}(\mathrm{x}) = \int \mu(\mathrm{s})/\mathrm{F}(\mathrm{s}) \triangleq \sup_{x=\mathrm{F}(\mathrm{s})} \mu(\mathrm{s}) = \mu(\mathrm{F}^{-1}(\mathrm{x})) \tag{13}$$

$$\Pi_{\mathrm{F}}(\mathrm{x}) = \int \Pi(\mathrm{s})/\mathrm{F}(\mathrm{s}) \triangleq \sup_{x=\mathrm{F}(\mathrm{s})} \Pi(\mathrm{s}) = \Pi(\mathrm{F}^{-1}(\mathrm{x})) \tag{14}$$

Here, we obtain fuzzy event probabilities by Equations (15) and (16) and find the larger of the two to discriminate between them.

$$\mathrm{P}_{\mathrm{T}}(\mathrm{F}_3) = \int \mu_{\mathrm{T}}(\mathrm{x}) \times \Pi_{\mathrm{T}}(\mathrm{x})\,\mathrm{dx} \tag{15}$$

$$\mathrm{P}_{\mathrm{F}}(\mathrm{F}_3) = \int \mu_{\mathrm{F}}(\mathrm{x}) \times \Pi_{\mathrm{F}}(\mathrm{x})\,\mathrm{dx} \tag{16}$$

We take the membership functions representing the true and false states to be inversely related, as in utility and loss functions, and following addition of the orthogonality condition we then maximize the probability representing the true state and minimize the probability representing the false state. By solving this two-objective programming problem, expressed as Equation (17), we find the indiscriminate probability, which at its largest is taken as the indiscriminate state.

$$\begin{aligned} &\max_{x}\ \mu_{\mathrm{T}}(\mathrm{x}) \times \Pi_{\mathrm{T}}(\mathrm{x}) \\ &\min_{x}\ \mu_{\mathrm{F}}(x') \times \Pi_{\mathrm{F}}(x') \\ &\text{s.t.} \\ &x + x' = 1 \end{aligned} \tag{17}$$

Conclusion

In this paper, we have demonstrated two approaches to the optical illusion state discrimination problem. The difference of the membership functions representing true and false states from utility （loss） functions holds even where the orthogonality condition is not fulfilled. In cases where both T（s） = 0 and F（s） = 0 and the fuzzy events thus represent no real state, they accordingly may be taken to represent a state of visual hallucination, and we plan further investigation in this area.

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