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Decision Method in Type-2 Fuzzy Events under Fuzzy Observed Information

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Abstract

Hori (né Uemura) et al. formulated the fuzzy-Bayes decision rule extension of the Wald decision function to fuzzy OR- and AND-connectives with multiple subjective distributions. This decision rule maps and transforms a state of nature to fuzzy events. In this context, the map of subjective distributions is called the subjective possibility distribution and the map of utility functions is called the fuzzy utility function. The subjective possibility distribution represents the possibility of possibilities and the fuzzy utility function represents the utility of utilities, and these are OR type-2 fuzzy. This definition differs from the definition of ordinary type-2 fuzzy sets. We further incorporate the concept of time and show that so long as the state of nature follows a Markov process, the fuzzy events thus manifested also represent a Markov process. As a Monte Carlo simulation with a fuzzy transitional matrix, however, it is marked by a repeating cycle of elimination, inversion, and restoration. In the present paper, in this fuzzy-Bayes decision rule we extend the fuzzy events to type-2 fuzzy and then to pseudo OR type-3 fuzzy, and report the formulation of the Markov decision process in type-2 fuzzy events incorporating the concept of time.

Keywords: type-2 fuzzy event, fuzzy-Bayes decision rule, Markov process, fuzzy information quantity, possibility measure, α-level cut, multi-objective programming .

INTRODUCTION

Tanaka et al. [1979] formulated the fuzzy-Bayes decision rule as an extension of Wald's subjective modification [Wald 1950] to fuzzy events by integral transformation based on expected utility maximization theory. Uemura, 1991], Uemura and Sakawa [1993], and Hori and Matsumoto [2016, 2017] formulated the fuzzy-Bayes decision rule extension of the Wald decision function to fuzzy OR- and AND-connectives with multiple subjective distributions. The decision rule is applied after mapping and transformation of the state of nature to fuzzy events, and by mapping of subjective distributions, utility functions, or other fuzzy functions to fuzzy events becomes OR type-2 fuzzy. Hori and Matsumoto [2016, 2017] further incorporated the Markov time concept into states of nature, and derived the Markov process and Markov decision process in fuzzy events. This is a natural extension of the Wald decision function to stochastic processes. The fuzzy events emergent from the state of nature also comprise a Markov process with a fuzzy transition matrix and in Monte Carlo simulations repeat the cycle of elimination, inversion, and restoration. Hori and Matsumoto [2017] went on to present an illustrative adaptation of these fuzzy-Bayes decision rules as a method for determining an optical illusion state. In the present paper, we extend the fuzzy events to type-2 fuzzy and formulate a pseudo OR type-3 fuzzy decision rule. We next incorporate the Markov time concept into the state of nature and formulate the Markov decision process in type-2 fuzzy events. Lastly, we consider the problem of anomalous values in critical regions, which is commonly encountered in traditional stochastic processing and is amenable to OR type-2 fuzzy operations, and the problem of missing-value processing (e.g., outlier processing), which may be amenable to pseudo OR type-3 fuzzy operations. In this paper, we proceed with the discussion of the assumption that the type-2 fuzzy events are identified from the observed information, as allowing envelopment of the data in a dome-shaped distribution obtained by a fuzzy polynomial regression or other such model.

Decision method-1 in type-2 fuzzy events

We assume that the subjective distribution π(S) and utility function U(S|D) have been identified by the decision maker by lottery, for state of nature S, where D denotes actions. Using a fuzzy polynomial regression model to envelop the observed information, we denote the upper, central, and lower fuzzy events as μF1(S), μF2(S), and μF3(S), respectively. We further assume that the fuzzy risk for the state of nature is μR1(S), μ R2(S), or μR3(S) when the decision maker is respectively risk-tolerant, risk-neutral, or risk-averse. Applying Zadeh's extension principle of mapping, we derive three types of subjective possibility distribution and fuzzy utility functions as [2016, 2017]

$$\mu_{F_1}(\Pi^{-1}(S)),\ \mu_{F_2}(\Pi^{-1}(S)),\ \mu_{F_3}(\Pi^{-1}(S)) \quad (1)$$

$$\mu_{F_1}(U^{-1}(S|D)),\ \mu_{F_2}(U^{-1}(S|D)),\ \mu_{F_3}(U^{-1}(S|D)) \quad (2)$$

The upper, central, and lower levels respectively represent risk-tolerant, risk-neutral, and risk-averse fuzzy events, and we accordingly propose deriving the fuzzy information quantities with respect to risk, defining the possibility measures in the integrated type-2 fuzzy events and taking the maximum as the optimal action as follows.

$$\begin{aligned} W_{R_1} &\triangleq \max_S \mu_{R_1}(S)\log\mu_{R_1}(S) \\ W_{R_2} &\triangleq \max_S \mu_{R_2}(S)\log\mu_{R_2}(S) \\ W_{R_3} &\triangleq \max_S \mu_{R_3}(S)\log\mu_{R_3}(S) \end{aligned} \quad (3)$$

$$\begin{aligned} \Pi_D \triangleq\ & W_{R_1}\cdot\max_S \mu_{F_1}(\Pi^{-1}(S))\cdot\mu_{F_1}(U^{-1}(S|D)) + W_{R_2}\cdot\int\mu_{F_2}(\Pi^{-1}(S))\cdot\mu_{F_2}(U^{-1}(S|D))\,ds \\ & + W_{R_3}\cdot\max_S \min(\mu_{F_3}(\Pi^{-1}(S)),\ \mu_{F_3}(U^{-1}(S|D)))\quad D^* = \max_D \Pi_D \end{aligned} \quad (4)$$

Decision method-2 in Type-2 fuzzy events

As an alternative method for deriving the integrated possibility measure for fuzzy events identified as dome-shaped distribution from the observed information, we perform an α-level cut of the fuzzy events. Since the upper and lower fuzzy events are respectively risk-tolerant and risk-averse, wecan then formulate the decision rule as shown in equation 5. As a two-objective programming problem, this yields innumerable Pareto solutions, and we therefore incorporate a fuzzy goal such as equation 6, derive the integrated possibility measures, and take the result with the maximum possibility measure as the optimal action

$$\begin{cases} \max_{\alpha} \max_{S} \mu_{F1\alpha}(\Pi^{-1}(S)) \cdot \mu_{F1\alpha}(U^{-1}(S|D)) \\ \min_{\alpha} \max_{S} \mu_{F3\alpha}(\Pi^{-1}(S)) \cdot \mu_{F3\alpha}(U^{-1}(S|D)) \end{cases} \quad (5)$$

$$\begin{cases} \max_{S} \mu_{F1\alpha=0}(\Pi^{-1}(S)) \cdot \mu_{F1\alpha=1}(U^{-1}(S|D)) \\ \max_{S} \mu_{F1\alpha=1}(\Pi^{-1}(S)) \cdot \mu_{F1\alpha=0}(U^{-1}(S|D)) \end{cases}$$

$$\begin{cases} \max_{S} \mu_{F3\alpha=1}(\Pi^{-1}(S)) \cdot \mu_{F3\alpha=0}(U^{-1}(S|D)) \\ \max_{S} \mu_{F3\alpha=0}(\Pi^{-1}(S)) \cdot \mu_{F3\alpha=1}(U^{-1}(S|D)) \end{cases} \quad (6)$$

Markov decision process in type-2 fuzzy events with incorporated fuzzy risk

Hori and Matsumoto [2016, 2017] proved that if the state of nature can be assumed to follow the Markov process of a transition matrix L(t, χt), then the fuzzy events also follow a Markov process with a fuzzy transition matrix. The fuzzy Markov processes of the upper, central, and lower levels are then

$$L^{-1}(t, \mu_{F1t}(\chi_t)), L^{-1}(t, \mu_{F2t}(\chi_t)), \quad L^{-1}(t, \mu_{F3t}(\chi_t)) \quad (7)$$

With the fuzzy event membership function μ4t(χt) as in equation (8) after weighting for fuzzy information quantity, the Markov process in the type-2 fuzzy events can be expressed as in Equation (9).

$$\mu_{4t}(\chi_t) \triangleq W_{R1} \cdot \mu_{F1t}(\chi_t) + W_{R2} \cdot \mu_{F2t}(\chi_t) + W_{R3} \cdot \mu_{F3t}(\chi_t) \quad (8)$$

$$D_t = L^{-1}(t, \mu_{4t}(\chi_t)) \quad (9)$$

We next incorporate the decision process At with one decision for each process and denote as UtAt(χt) the utility function weighted for fuzzy information quantity. The fuzzy utility function is then derived as equation 10 by the mapping extension principle, and its Markov decision process is expressed as in equation 11.

$$F_{U_tA_t}(\chi_t) = \mu_{4t}(U^{-1}{}_t A_t(\chi_t)) \quad (10)$$

$$D_{FU_t} = L^{-1}(t, F_{U_tA_t}(\chi_t)) \quad (11)$$

We calculate the possibility measure using the max-product operation, and take the result with the maximum possibility measure as the optimal action.

$$\Pi_{D_t} = \max_{\chi_t} D_t \times D_{FU_t}$$

$$D^*_t \triangleq \max_{D_t} \Pi_{D_t} \qquad (12)$$

Conclusion

In this paper, we have described the implementation of two approaches for the decision method in type-2 fuzzy events. In one, we formulate a new incorporation of a fuzzy risk concept relative to the state of nature, calculate the fuzzy information quantity, and apply a weighted possibility measure. In the other, we formulate the application of a direct α-level cut and a method for multi-objective programming. In application, the first approach is simpler in cases where risk-tolerant, risk-neutral, and risk-averse fuzzy risks can be readily distinguished. We have also described the formulation of a Markov decision process in type-2 fuzzy events with incorporated fuzzy risk. The method for fuzzy risk specification remains for further study. Developments are also planned for extension to no-data problems, for which the main requirement will be an effective method for type-2 fuzzy events specification.

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