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Markov decision process in fuzzy events based on the mapping extension principle

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Abstract

It has been shown by Hori and Matsumoto that if a state of nature constitutes an ergodic Markov process, then the fuzzy events manifested by that state of nature also follow a Markov process. Here, we first perform mapping and transformation in accordance with their membership function of the subjective distribution and the utility function that occur in the no-data problem in fuzzy events with Markov transition in order to formulate their Markov decision processes using the max-product method. We note that this series flow is a natural extension of the Wald decision function to a stochastic process. Next, taking the subjective distribution and the utility function as fuzzy functions and assuming that the state of nature is mapped and transformed to subjectivity by the subjective distribution and to utility by the utility function, we show that the subjectivity and the utility also follow Markov processes. We conclude by proposing a Markov decision process in which the subjectivity and the utility in fuzzy events transition according to Markov processes and are elements that constitute the transition matrices that define the Markov processes, and the max-product method is applied.

Keywords. fuzzy events, subjective distribution, utility function, extension principle, Markov process, Markov decision process

Introduction

Tanaka et al. [1979] formulated the fuzzy Bayes decision rule by integral transformation based on expected utility maximization theory as an extension of Wald's subjective modification [1950] to fuzzy events. Uemura [1991], Uemura and Sakawa [1993], and Hori and Matsumoto [1993, 2016] formulated the extension of the Wald decision function to the fuzzy Bayes decision rule including multiple subjective functions and fuzzy OR and AND-connectives. This decision rule is applied after mapping and transformation of the state of nature to fuzzy events, and by mapping fuzzy functions such as a subjective distribution and utility function to fuzzy events, they become OR type-2 fuzzy. Hori and Matsumoto [2016] further incorporated the concept of Markov-type time into the state of nature and derived the Markov process and Markov decision process in fuzzy events. It thus represents a natural extension of the Wald decision function to stochastic process theory, and the fuzzy events manifesting the state of nature also follow a Markov process with a fuzzy transition matrix and as a result of Monte Carlo simulation repeat a cycle of elimination, inversion, and restoration. Hori and Matsumoto [2017] then illustrated the application of the fuzzy Bayes decision rules with a proposed method for discrimination of a state of illusion. In the present paper, we first formulate Markov decision processes in which the subjective distribution and the utility function in the no-data problem that undergo ergodic Markov transition by mapping and transformation of these fuzzy events with their membership function and applying the max-product method. We note this serial flow is a natural extension of the Wald decision function to a stochastic process. Next, taking the subjective distribution and the utility function as fuzzy functions, if we assume that subjectivity is the entity that maps and transforms the state of nature according to the subjective distribution and utility is the entity that maps and transforms the state of nature according to the utility function, then the subjectivity and the utility themselves also follow Markov processes. Lastly, we propose Markov decision processes in which the subjectivity and the utility in fuzzy events transition according to Markov processes and are elements constituting transition matrices that define Markov processes, and the max-product method is applied.

Markov decision process in fuzzy events using the max-product method

It has been shown that if the state of nature Dt follows the Markov process of transition matrix L according to Equation （1）, then the fuzzy events Ft which are its manifestations follow the Markov process expressed in Equation 2 [Hori and Matsumoto, 2016 and 2017], whereμF（t χt） is the fuzzy event membership function and L-1 is the inverse matrix of the transition matrix.

$$D_t = L(t, \chi_t) \qquad (1)$$

$$F_t = L^{-1}(t, \mu_{F_t}(\chi_t)) \qquad (2)$$

In the no-data problem, if it is assumed that the subjective distribution Π^t and utility function Ut,D are identified sequentially by decision-maker lottery, then after mapping and transformation, the subjective possibility distribution and the fuzzy utility function in fuzzy events can be separately derived using the extension principle as

$$\Pi_t \triangleq \sup_{\{y_t=\Pi(\chi_t)\}} L^{-1}(t, \mu_{F_t}(\chi_t)) = L^{-1}(t, \mu_{F_t}(\Pi^{-1}(y_t))) \qquad (3)$$

$$U_{t,D} \triangleq \sup_{\{y_t=U(\chi_t|D)\}} L^{-1}(t, \mu_{F_t}(\chi_t)) = L^{-1}(t, \mu_{F_t}(U^{-1}(y_t|D))) \qquad (4)$$

where Π^{-1} is the inverse function of the subjective distribution and U^{-1} is the inverse function of the utility function. Note also that it has been shown using subjective Bayes theory and utility function theory that the extension principle is effective [Hori and Matsumoto, 2016]. It has been demonstrated that fuzzy inference is appropriate for high-risk high-return decision problems when used in concert with the max-product method [Hori and Matsumoto, 2016 and 2017]. The possibility measure ΠΠt found by the max-product method is shown in Equation （5）. We next propose the Markov decision process with weighting by the fuzzy events by using this possibility measure and taking the maximum as the optimum action as in Equation （6）.

$$\Pi\Pi_t \triangleq \max\ \Pi_t \times U_{t,D} = L^{-1}(t, \max\{\mu_{F_t}(\Pi^{-1}(y_t)) \times \mu_{F_t}(U^{-1}(y_t|D))\}) \qquad (5)$$

$$D^* = \max_{D}\ \Pi\Pi_t \times F_t \qquad (6)$$

Markov decision process in fuzzy events based on the extension principle

The subjective distribution and the utility function in the no-data problem are determined by decision-maker lottery based on certainty equivalence, and can therefore be regarded as piecewise linear fuzzy functions with fuzzy OR-connectives. Accordingly, subjectivity may be regarded as mapping and transformation of the state of nature by the subjective distribution and utility may be regarded as mapping and transformation of the state of nature by the utility function. Applying the mapping extension principle, the subjectivity and the utility thus both follow Markov processes, as

$$\Pi_t = L^{-1}(t, \Pi_t(\chi t)) \qquad (7)$$

$$U_{t,D} = L^{-1}(t, U_{t,D}(\chi t)) \qquad (8)$$

We next formulate the Markov decision process in fuzzy events. Subjectivity FΠt in fuzzy events can be regarded as subjectivity mapped and transformed by the membership function of fuzzy events, which can be derived as in Equation （9） by the mapping extension principle, and utility FUt,D is similarly given by Equation （10）.

$$F\Pi_t \triangleq \sup_{\{y_t = L^{-1}(t, \Pi_t(x_t))\}} L^{-1}(t, \mu_{F_t}(\chi t)) = L^{-1}(t, L^{-1}(t, \mu_{F_t}(\Pi_t^{-1}(y_t)))) \qquad (9)$$

$$FU_{t,D} \triangleq \sup_{\{y_t = L^{-1}(t, U_{t,D}(x_t))\}} L^{-1}(t, \mu_{F_t}(\chi t)) = L^{-1}(t, L^{-1}(t, \mu_{F_t}(U_{t,D}^{-1}(y_t)))) \qquad (10)$$

We apply the max-product method and formulate the Markov decision process in fuzzy events as

$$D^* \triangleq \max_{D} \max F\Pi_t \times FU_{t,D} \qquad (11)$$

Conclusion

In this paper, we have focused on the high-risk high-return decision no-data problem and derived the Markov decision process by using the max-product method, and further derived the fuzzy Markov decision process by taking the subjective distribution and the utility function as fuzzy functions and applying the mapping extension principle. Note that decision problems case-classified as other decision-maker risk types can be immediately derived by replacing the possibility-measure operation with the integral transformation or the max-min operation. For integration of these operations, refer to reference [Hori et al., 2017].

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